$\boxed{18}$ Calcular la distancia que hay entre las rectas

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$$r \equiv \begin{cases} x = -2+3\lambda \\ y = 2\lambda \\ z = 1+\lambda \end{cases} \lambda \in \mathbb{R} \qquad s \equiv \begin{cases} x = 1-\mu \\ y = 5\mu \\ z = -2+\mu \end{cases} \mu \in \mathbb{R}$$

a) $\vec{PQ}$ b) vol del paralel. $\vec{PQ}, \vec{u_r}, \vec{u_s}$ c) $d(r,s)$

a) $r \equiv \begin{cases} \vec{u_r}(3,2,1) \\ P(-2,0,1) \end{cases}$ $\qquad s \equiv \begin{cases} \vec{u_s}(-1,5,1) \\ Q(1,0,-2) \end{cases}$

$\vec{PQ}\ (3,0,-3)$

b) $V = |\vec{PQ} \cdot (\vec{u_r} \wedge \vec{u_s})| = \begin{vmatrix} 3 & 0 & -3 \\ 3 & 2 & 1 \\ 1 & 0 & -2 \end{vmatrix} = |-60| = 60\ u^3$

c) $A = |\vec{u_r} \wedge \vec{u_s}| = \left| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 1 \\ -1 & 5 & 1 \end{vmatrix} \right| = |(-3,-4,17)| = \sqrt{314}\ u^2$

d) $d(r,s) = \dfrac{V}{A} = \dfrac{|\vec{PQ} \cdot (\vec{u_r} \wedge \vec{u_s})|}{|\vec{u_r} \wedge \vec{u_s}|} = \dfrac{60}{\sqrt{314}} = \dfrac{60\sqrt{314}}{314} =$

$= \boxed{\dfrac{30\sqrt{314}}{157}\ u}$

$\boxed{23}$ a) Hallar la ecuación del plano Π perpendicular a la recta $r \equiv \dfrac{x+3}{2} = \dfrac{y-4}{3} = \dfrac{z}{4}$ y que pasa por el pto $P(-1,1,0)$.

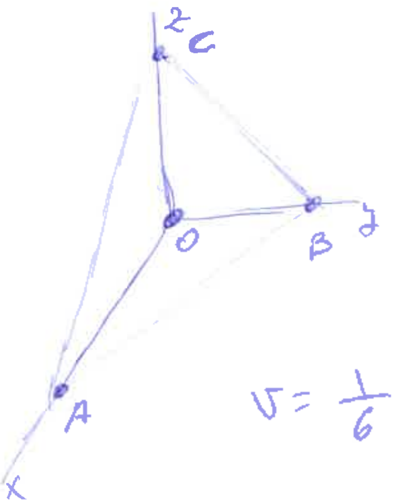
b) Calcular el vol. de la figura limitada por Π y los 3 planos coordenados.

a) $\Pi \perp r \Leftrightarrow \vec{n}_{\Pi} = \vec{u}_r \; (2,3,4) \Rightarrow 2x+3y+4z+D=0$

pasa por $P(-1,1,0) \Rightarrow -2+3+D=0 \Rightarrow D=-1$

$\Rightarrow \underline{\Pi \equiv 2x+3y+4z-1=0}$

b)



$A \to y=0,\ z=0 \Rightarrow x = 1/2 \quad A(1/2, 0, 0)$

$B \to x=0,\ z=0 \Rightarrow y = 1/3 \quad B(0, 1/3, 0)$

$C \to x=0,\ y=0 \Rightarrow z = 1/4 \quad C(0, 0, 1/4)$

Es el volumen de un tetraedro.

$V = \frac{1}{6} \left| \vec{OA} \cdot (\vec{OB} \wedge \vec{OC}) \right| = \frac{1}{6} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} = \frac{1}{144} \; u^3$

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n.º 29 Hallar los pts de la recta $r \equiv x-1 = y+2 = z$ que equidistan de los planos $\alpha \equiv 4x-3y-1=0$ $\beta \equiv 3x+4y-1=0$

$P_r(1+\lambda, -2+\lambda, \lambda)$

$$d(P_r, \alpha) = d(P_r, \beta) \Rightarrow \frac{|4(1+\lambda) - 3(-2+\lambda) - 1|}{\cancel{\sqrt{16+9}}} = \frac{|3(1+\lambda) + 4(-2+\lambda) - 1|}{\cancel{\sqrt{9+16}}}$$

$$|4 + 4\lambda + 6 - 3\lambda - 1| = |3 + 3\lambda - 8 + 4\lambda - 1|$$

$$|\lambda + 9| = |7\lambda - 6|$$

- $\lambda + 9 = 7\lambda - 6 \Rightarrow 6\lambda = 15 \Rightarrow \lambda = 15/6 = 5/2$

  $P_r(1 + 5/2, -2 + 5/2, 5/2) \rightarrow P_r(7/2, 1/2, 5/2)$

- $\lambda + 9 = 6 - 7\lambda \Rightarrow 8\lambda = -3 \Rightarrow \lambda = -3/8$

  $P_r(5/8, -19/8, -3/8)$

45 Un cuadrado tiene uno de sus lados sobre la recta $r \equiv \begin{cases} 3x+2y+2z=0 \\ x-2y+2z=0 \end{cases}$ y otro sobre $s \equiv \frac{x-3}{2} = \frac{y-1}{-1} = \frac{z+5}{-2}$

a) Calcular el área del cuadrado

b) Si uno de los vértices del cuadrado es $(0,0,0)$ ¿Cuál es el otro vértice situado sobre la recta r? ¿Y sobre s? (A)

$$\vec{u}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 2 & 2 \\ 1 & -2 & 2 \end{vmatrix} = 8\vec{i} - \vec{j}\cdot 4 + \vec{k}\cdot(-8) \approx (2,-1,-2) = \vec{u}_s \Rightarrow r \parallel s \text{ ó } r = s$$

$P_s(3,1,-5) \notin r \Rightarrow r \parallel s$

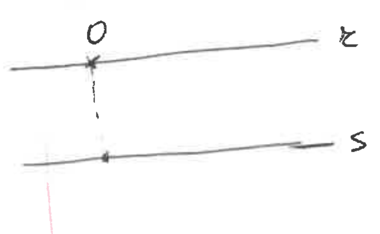
a) $A = d(r,s)^2 \Rightarrow$ $d(r,s) = d(P_r, s) = \dfrac{|\vec{P_rP_s} \times \vec{u}_s|}{|\vec{u}_s|} = \dfrac{\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & 1 & -5 \\ 2 & -1 & -2 \end{vmatrix}\right|}{\sqrt{4+1+4}} =$

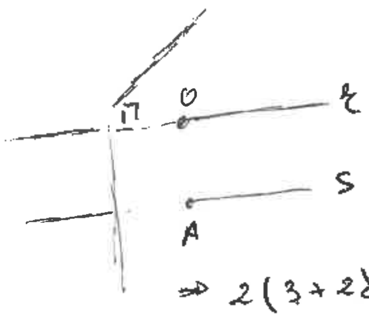
$P_r(0,0,0) \Rightarrow \vec{P_rP_s}\ (3,1,-5)$

$$= \frac{|(-7,-4,-5)|}{3} = \frac{\sqrt{49+16+25}}{3} = \sqrt{10}\ u$$

$$A = d(r,s)^2 = (\sqrt{10})^2 = \underline{10\ u^2}$$

b)





$\begin{cases} \Pi \perp r \Rightarrow \vec{u}_r(2,-1,-2) = \vec{n} \\ O \in \Pi \end{cases}$

$\Pi \equiv 2x - y - 2z = 0$

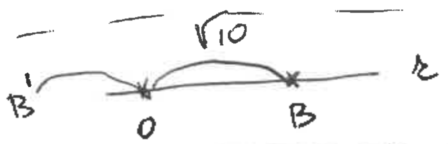
$A = \Pi \cap s \Rightarrow$

$\Rightarrow 2(3+2\lambda) - (1-\lambda) - 2(-5-2\lambda) = 0$

$9\lambda + 15 = 0$

$\boxed{\lambda = -5}$

$\Rightarrow A\left(-\frac{1}{3}, \frac{8}{3}, -\frac{5}{3}\right)$



$\Rightarrow$ $r \equiv \begin{cases} x = 2\lambda \\ y = -\lambda \\ z = -2\lambda \end{cases} \quad \lambda \in \mathbb{R}$

$d(O,B) = \sqrt{(2\lambda)^2 + \lambda^2 + (2\lambda)^2} = \sqrt{9\lambda^2} = \sqrt{10}$

$\Rightarrow 9\lambda^2 = 10 \Rightarrow \lambda = \pm\sqrt{\frac{10}{9}} = \pm\frac{\sqrt{10}}{3} \Rightarrow$

$\lambda = \frac{\sqrt{10}}{3}$ $\quad B\left(\frac{2\sqrt{10}}{3}, -\frac{\sqrt{10}}{3}, -\frac{2\sqrt{10}}{3}\right)$

$\lambda = -\frac{\sqrt{10}}{3}$ $\quad B'\left(-\frac{2\sqrt{10}}{3}, \frac{\sqrt{10}}{3}, \frac{2\sqrt{10}}{3}\right)$

dos soluciones

$r \subset \pi$ | n° 30 | Pg. 196

$\pi \perp \sigma$

$r \equiv \begin{cases} x+y-z+1=0 \\ x+2y+z=0 \end{cases}$

$\sigma \equiv 2x-y+3z+1=0$

$\pi \supset r \iff \vec{u}_r = \vec{u}_\pi \quad P_r \in \pi$

$\pi \perp \sigma \iff \vec{n}_\alpha = \vec{v}_\pi$

$$\vec{u}_r = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & 1 & -1 \\ 1 & 2 & 1 \end{vmatrix} = 3\vec{i} - 2\vec{j} + k$$

$n_\alpha = (2, -1, 3)$ $\qquad P_r\left(-\frac{1}{2}, 0, +\frac{1}{2}\right)$

$$\pi \equiv \begin{vmatrix} x+\frac{1}{2} & y & z-\frac{1}{2} \\ 3 & -2 & 1 \\ 2 & -1 & 3 \end{vmatrix} = 0 \Rightarrow -5x-7y+z-3=0$$

$\boxed{5x+7y-z+3=0}$

b) $s = \pi \cap \sigma$

$$\vec{u}_s = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 5 & 7 & -1 \\ 2 & -1 & 3 \end{vmatrix} = 20\vec{i} - 17\vec{j} - 19k \quad (20, -17, -19)$$

¿P_s? $\begin{cases} 5x+7y-z+3=0 \\ 2x-y+3z+1=0 \end{cases} \Rightarrow x=0 \begin{cases} 7y-z=-3 \\ -y+3z=-1 \end{cases} \Rightarrow \begin{cases} y=-\frac{1}{2} \\ z=-\frac{1}{2} \end{cases} \quad P_s\left(0, -\frac{1}{2}, -\frac{1}{2}\right)$

$$s \equiv \begin{cases} x = 20\lambda \\ y = -\frac{1}{2} - 17\lambda \\ z = -\frac{1}{2} - 19\lambda \end{cases} \quad \lambda \in \mathbb{R}$$

c) $\alpha = \angle(r, \sigma) \Rightarrow \operatorname{sen} \alpha = \dfrac{|\vec{u}_r \cdot \vec{n}_\sigma|}{|\vec{u}_r||\vec{n}_\sigma|} = \dfrac{|(3,-2,1)\cdot(2,-1,3)|}{\sqrt{9+4+1}\sqrt{4+1+9}} = \dfrac{11}{14}$

$\Rightarrow \alpha = \operatorname{arc\,sen} \frac{11}{14} = 51°\,47'\,12{,}44''$

$\boxed{31}$ $r = \begin{cases} x - 2z + 3 = 0 \\ y - z - 4 = 0 \end{cases}$

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$\pi \equiv x + 2y + 3z - 1 = 0$

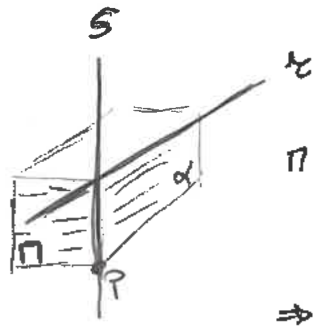
$s \equiv \begin{cases} s \subset \pi & \longrightarrow \vec{u}_s \perp \vec{n}_\pi \\ P(2,1,-1) & \\ s \perp r & \longrightarrow \vec{u}_s \perp \vec{u}_r \end{cases}$ $\Big\rangle$ $\vec{u}_s = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & 2 & 3 \\ +2 & 1 & 1 \end{vmatrix} = -\vec{i} + 5\vec{j} + 3k$

$u_r = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & 0 & -2 \\ 0 & 1 & -1 \end{vmatrix} = +2\vec{j} + \vec{i} + k$

$\vec{u}_s = \begin{cases} x = 2 + \lambda \\ y = 1 + 5\lambda \\ z = -1 + 3\lambda \end{cases}$ $\lambda \in \mathbb{R}$

34 Hallar la ecuac. de la recta que pasa por el pto $(1,2,1)$ y corta $\perp$ a la recta $r \equiv \begin{cases} x - y - z = 1 \\ x + z = 2 \end{cases}$

I



$s = \pi \cap \alpha$ con $\pi \perp r$ y $P \in \pi$

$r \subset \alpha$ y $P \in \alpha$

π? $\pi \perp r \Leftrightarrow \vec{n}_\pi = \vec{u}_r = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & -1 & -1 \\ 1 & 0 & 1 \end{vmatrix} = -\vec{i} - 2\vec{j} + \vec{k} \approx (1,2,-$

(1)

$\Rightarrow \pi \equiv (x-1) + 2(y-2) - (z-1) = 0$

$\underline{\pi \equiv x + 2y - z - 4 = 0}$

α? $r \subset \alpha \Leftrightarrow \vec{u}_r = \vec{u}_\alpha = (1,2,-1)$

$\overrightarrow{PP_r} = \vec{v}_\alpha$

$P_r \to z = 0 \Rightarrow x = 2 \Rightarrow y = 1 \Rightarrow P_r(2,1,0)$

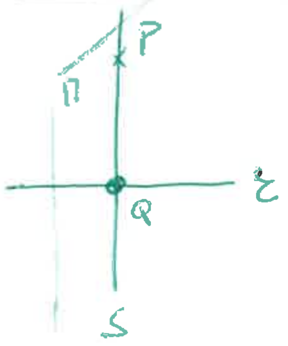
$\overrightarrow{PP_r}\ (1,-1,-1) = \vec{v}_\alpha$

$\alpha \equiv \begin{cases} \vec{u}_r\ (1,2,-1) \\ \overrightarrow{PP_r}\ (1,-1,-1) \\ P\ (1,2,1) \end{cases} \Rightarrow \alpha \equiv \begin{vmatrix} x-1 & y-2 & z-1 \\ 1 & 2 & -1 \\ 1 & -1 & -1 \end{vmatrix} = 0$

$\alpha \equiv -3x - 3z + 6 = 0 \to \underline{\alpha \equiv x + z - 2 = 0}$

Luego $s \equiv \begin{cases} x + 2y - z - 4 = 0 \\ x + z - 2 = 0 \end{cases}$

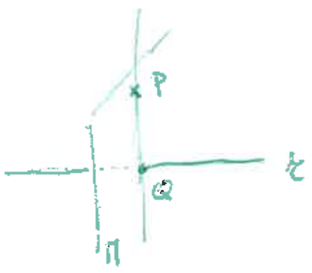
II



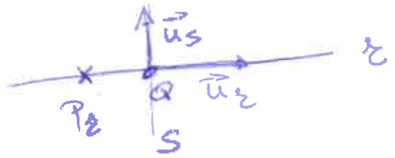
a) Hallamos $\pi \perp r$ y que contenga a P

b) " $\pi \cap r = Q$

c) $s \equiv \begin{cases} \overrightarrow{PQ} \\ \text{Pasa por } P(1,2,1) \end{cases}$



III) por ptos genéricos.



Ponemos r en paramétricas

$z=\lambda \Rightarrow x = 2-\lambda \quad \wedge \quad y = x-z-1 = 2-\lambda-\lambda-1 = 1-2\lambda$

$$r \equiv \begin{cases} x = 2-\lambda \\ y = 1-2\lambda \\ z = \lambda \end{cases} \quad \lambda \in \mathbb{R} \Rightarrow P_r(2-\lambda,\ 1-2\lambda,\ \lambda)$$

($\vec{u}_s = \overrightarrow{P_rP}$ pero tiene que ser $\perp$ a $\vec{u}_r$)

$\odot$ $\overrightarrow{P_rP} \perp \vec{u}_r$

$\overrightarrow{P_rP}(1-\lambda,\ -1-2\lambda,\ \lambda-1)$ $\Big\}$ $\overrightarrow{P_rP} \cdot \vec{u}_r = 0$

(1) $\vec{u}_r(1,2,-1)$

$$(1-\lambda,\ -1-2\lambda,\ \lambda-1)\cdot(1,2,-1) = 0 \Rightarrow \underline{\underline{\lambda = 0}}$$

$\Rightarrow P_r(2,1,0)$

Luego $\vec{u}_s = \overrightarrow{PP_r}(1,-1,-1)$

$$s \equiv \begin{cases} \vec{u}_s(1,-1,-1) \\ P(1,2,1) \end{cases} \equiv \begin{cases} x = 1+\mu \\ y = 2-\mu \\ z = 1-\mu \end{cases} \quad \underline{\underline{\mu \in \mathbb{R}}}$$

$\boxed{36}$ $P \in r$ $\quad r \equiv \frac{x-1}{2} = \frac{y+1}{1} = \frac{z}{3}$ $\quad r \equiv \begin{cases} x = 1 + 2t \\ y = -1 + t \\ z = 3t \end{cases} \quad t \in \mathbb{R}$

$\alpha \equiv x + y + z + 3 = 0$ $\quad P(1+2t, -1-t, 3t)$

$\beta \equiv \begin{cases} x = -3 + \lambda \\ y = -\lambda + \mu \\ z = -6 + \mu \end{cases}$ $\quad \vec{n}_\beta = \begin{vmatrix} \vec{i} & \vec{j} & k \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{vmatrix} = -\vec{i} - \vec{j} + \vec{k}$ $\quad \vec{n}_\beta(1, 1, -1)$

$P_\beta = (-3, 0, -6)$

$\beta \equiv (x+3) + y - (z+6) = 0 \Rightarrow \beta \equiv x + y - z - 3 = 0$

$d(P, \alpha) = d(P, \beta)$

$d(P, \alpha) = \left| \frac{\cancel{1} + 2t - \cancel{1} + t + 3t + 3}{\sqrt{3}} \right| = \left| \frac{6t + 3}{\sqrt{3}} \right|$ $\Rightarrow |6t+3| = 3$

$d(P, \beta) = \left| \frac{\cancel{1} + 2t - \cancel{1} + t - 3t - 3}{\sqrt{3}} \right| = \left| \frac{-3}{\sqrt{3}} \right|$

$\therefore$ ⊙ $6t + 3 = -3 \Rightarrow 6t = -6 \Rightarrow \boxed{t = -1}$ $\quad \boxed{P(-1, -2, -3)}$

⊙ $6t + 3 = 3 \Rightarrow t = 0$ $\quad \boxed{P(1, -1, 0)}$

$\boxed{40}$ ¿Π? $r \equiv \begin{cases} x = -1+3\lambda \\ y = 1+2\lambda \\ z = 2+\lambda \end{cases}$ $\lambda \in \mathbb{R}$ $\underline{r \subset \Pi} \Rightarrow \vec{u}_r \perp \vec{n}_\Pi$

$P_r \in \Pi$ $P_r(-1,1,2)$

$\underline{\Pi \perp \alpha \equiv 2x+y-3z+4=0} \Leftrightarrow \vec{v}_\Pi = \vec{n}_\alpha$

$$\Pi \equiv \begin{vmatrix} x+1 & y-1 & z-2 \\ 3 & 2 & 1 \\ 2 & 1 & -3 \end{vmatrix} = 0 \Leftrightarrow \boxed{\Pi \equiv 7x-11y+z+16=0}$$

$$\alpha - \angle(r, \Pi) \quad \operatorname{sen} \alpha = \frac{|\vec{n} \cdot \vec{u}|}{|\vec{n}|\,|\vec{u}|} = \frac{|(7,-11,1)(3,2,1)|}{\sqrt{7^2+11^2+1}\,\sqrt{9+4+1}} =$$

$$= \frac{|21-32+1|}{} \Rightarrow \alpha = 20^\circ 55' 29''$$

Hallar la ecuación del plano que contiene a la recta de ecuaciones paramétricas

$(-1+3\lambda,\ 1+2\lambda,\ 2+\lambda)$

y es perpendicular al plano $2x+y-3z+4=0$

Determina tb el ángulo formado por la recta y el plano dados.

$\boxed{38}$ a) Hallar la ecuac de la recta que corta $\perp$ a r y s

$$r \equiv \begin{cases} x = -3+\lambda \\ y = -2+5\lambda \\ z = 0 \end{cases} \lambda \in \mathbb{R} \quad , \quad s \equiv \begin{cases} x = 3 \\ y = -6+4\mu \\ z = 2+\mu \end{cases} \mu \in \mathbb{R}.$$

b) $d(r,s)$

a) P.R de r y s $\Rightarrow$ $r \equiv \begin{cases} \vec{u}_r(1,5,0) \\ P_r(-3,-2,0) \end{cases}$ $\quad s \equiv \begin{cases} \vec{u}_s(0,4,1) \\ P_s(3,-6,2) \end{cases}$ $\quad \overrightarrow{P_rP_s}(6,-4,2)$

$\vec{u}_r \not\parallel \vec{u}_s \rightarrow \frac{1}{0} \neq \frac{5}{4} \neq \frac{1}{0} \Rightarrow$ se cortan o cruzan.

rg $(\overrightarrow{P_rP_s}, \vec{u}_r, \vec{u}_s)$? $\quad \begin{vmatrix} 1 & 5 & 0 \\ 0 & 4 & 1 \\ 6 & -4 & 2 \end{vmatrix} = 42 \neq 0 \Rightarrow$ se cruzan.

$t \perp r$ y $t \perp s \Rightarrow \vec{u}_t = \vec{u}_r \wedge \vec{u}_s = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & 0 \\ 0 & 4 & 1 \end{vmatrix} = 5\vec{i} - \vec{j} + 4\vec{k} \equiv (5,-1,4)$

- Construimos π_1 t.q. $\begin{matrix} r \subset \pi_1 \\ t \parallel \pi_1 \end{matrix} \Rightarrow \pi_1 \equiv \begin{cases} \vec{u}_r(1,5,0) \\ P_r(-3,-2,0) \\ \vec{u}_t(5,-1,4) \end{cases} \Rightarrow \pi_1 \equiv \begin{vmatrix} x+3 & y+2 & z \\ 1 & 5 & 0 \\ 5 & -1 & 4 \end{vmatrix} = 0$

$\Rightarrow \pi_1 \equiv 10x - 2y - 13z + 26 = 0$

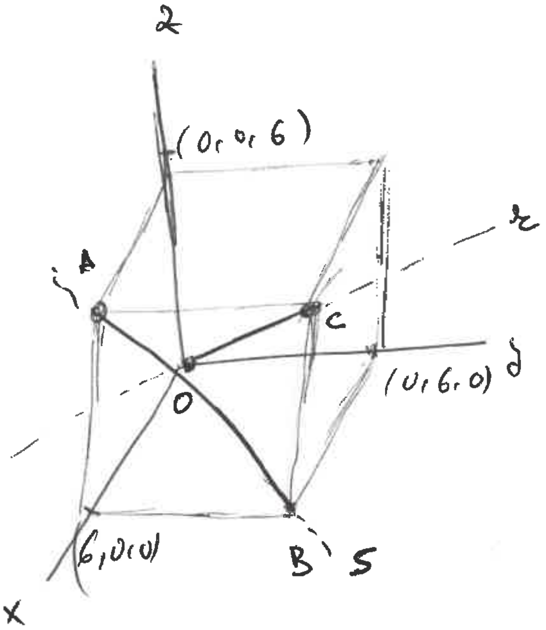
- Construimos π_2 t.q. $\begin{matrix} s \subset \pi_2 \\ t \parallel \pi_2 \end{matrix} \Rightarrow \pi_2 \equiv \begin{cases} \vec{u}_s(0,4,1) \\ P_s(3,-6,2) \\ \vec{u}_t(5,-1,4) \end{cases} \Rightarrow \pi_2 \equiv \begin{vmatrix} x-3 & y+6 & z-2 \\ 0 & 4 & 1 \\ 5 & -1 & 4 \end{vmatrix} = 0$

$\Rightarrow \pi_2 \equiv 17x + 5y - 20z + 19 = 0$

Luego $t \equiv \begin{cases} 10x - 2y - 13z + 26 = 0 \\ 17x + 5y - 20z + 19 = 0 \end{cases}$

b) $$d(r,s) = \frac{|\overrightarrow{P_rP_s}(\vec{u}_r \wedge \vec{u}_s)|}{|\vec{u}_r \wedge \vec{u}_s|} = \frac{\left| \begin{vmatrix} 1 & 5 & 0 \\ 0 & 4 & 1 \\ 6 & -4 & 2 \end{vmatrix} \right|}{\left| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 5 & 0 \\ 0 & 4 & 1 \end{vmatrix} \right|} = \frac{42}{\sqrt{25+1+16}} = \frac{42}{\sqrt{42}} = \boxed{\sqrt{42}\ u}$$

1401
pg. 196



$$r \equiv \begin{cases} O(0,0,0) \\ \vec{u}_r \neq \vec{OC} = (6,6,6) \neq (1,1,1) \end{cases} \Rightarrow r \equiv \begin{cases} x = \lambda \\ y = \lambda \\ z = \lambda \end{cases}$$

$$s \equiv \begin{cases} A(6,0,6) \\ \vec{u}_s = (\vec{OB}) = \end{cases} \Rightarrow s \equiv \begin{cases} x = 6 \\ y = \mu \\ z = 6 - \mu \end{cases}$$

$$\left.\begin{array}{l} A(6,0,6) \\ B(6,6,0) \end{array}\right\} \vec{DB}\ (0,6,-6) \neq \vec{u}_s(0,1,-1)$$

$$d(r,s) = \frac{|\vec{u}_r \cdot (\vec{u}_s \wedge \overset{\vec{OA}}{\vec{P_rP_s}})|}{|\vec{u}_r \wedge \vec{u}_s|} = \frac{\begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & -1 \\ 6 & 0 & 6 \end{vmatrix}}{\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix}\right|} = \frac{|6-6-6|}{\sqrt{4+1+1}} = \frac{6}{\sqrt{6}} = \sqrt{6}\ u$$

$$\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 1 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = -2\vec{i} + \vec{j} + \vec{k} \neq (-2,1,1)$$

42 $P' =$ simétrico de $P(1,2,3)$ respecto del plano $\alpha \equiv x-3y-2z+4$

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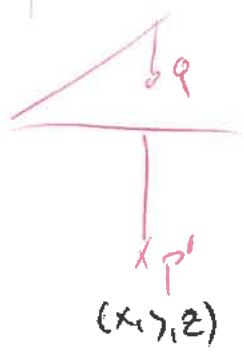
$r \perp \alpha$ pasando por P



i) $r \equiv (\vec{n}_\alpha, P(1,2,3)) = \begin{cases} x = 1+\lambda \\ y = 2-3\lambda \quad \lambda \in \mathbb{R} \\ z = 3-2\lambda \end{cases}$

ii) $Q = r \cap \alpha \iff 1+\lambda - 3(2-3\lambda) - 2(3-2\lambda) + 4 = 0$

$1+\lambda - 6 + 9\lambda - 6 + 4\lambda + 4 =$

$= 14\lambda - 7 = 0 \Rightarrow \lambda = 1/2$

$2 - 3/2$

$3 -$

$Q(3/2, 1/2, 2)$

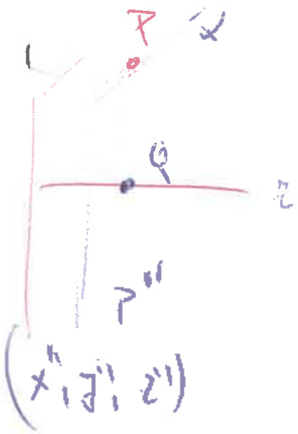
iii) Q pto medio de PP'

$$\left(\frac{3}{2}, \frac{1}{2}, 2\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z+3}{2}\right) \Rightarrow \begin{cases} x = 2 \\ y = -1 \\ z = 1 \end{cases}$$

$$\boxed{P'(2,-1,1)}$$

$P' =$ simétrico de $P(1,2,3)$ respecto de $r \equiv \begin{cases} x-y+3=0 \to \vec{n}_\alpha(1,-1,0) \\ 4x-z=0 \to \vec{n}_\alpha(4,0,-1) \end{cases}$

α plano $\perp$ a r que pasa por P



i) $\alpha \equiv \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & -1 & 0 \\ 4 & 0 & -1 \end{vmatrix} = 0 \Rightarrow \boxed{\alpha \equiv x+y+4z-15=0}$

ii) $Q = \alpha \cap r \equiv \begin{cases} x-y+3=0 \\ 4x-z=0 \\ x+y+4z-15=0 \end{cases} \Rightarrow \begin{cases} x = 2/3 \\ y = 11/3 \\ z = 8/3 \end{cases}$

iii) Q pto medio de PP''

$$\left(\frac{2}{3}, \frac{11}{3}, \frac{8}{3}\right) = \left(\frac{x+1}{2}, \frac{y+2}{2}, \frac{z+3}{2}\right) \Rightarrow \begin{cases} x'' = 1/3 \\ y'' = 16/3 \\ z'' = 7/3 \end{cases}$$

$$\boxed{P''(1/3, 16/3, 7/3)}$$

$\boxed{43}$ $d(P_r,\pi)=1/3$ $\quad r\equiv\begin{cases} x+y=0\\ x-z=0\end{cases}$ $\quad \pi\equiv 2x-y+2z+1=0$

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$O(0,0,0)$

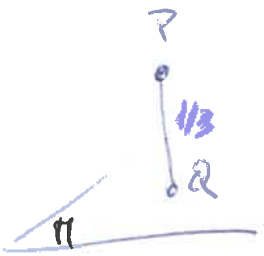
a) $r\equiv\begin{cases} x=\lambda\\ y=-\lambda\\ z=\lambda\end{cases}$ $\quad \underline{\lambda\in\mathbb{R}}$ $\quad P_r(\lambda,-\lambda,\lambda)$

$$d(P_r,\pi)=\frac{|2\lambda+\lambda+2\lambda+1|}{\sqrt{4+1+4}}=\frac{1}{3}\Rightarrow$$

$$\Rightarrow |5\lambda+1|=1 \Rightarrow \begin{cases} 5\lambda+1=1 \Rightarrow \lambda=0 \Rightarrow P_1(0,0,0)\\ 5\lambda+1=-1 \Rightarrow \lambda=-2/5 \quad P_2(-2/5,2/5,-2/5)\end{cases}$$

b) $P(0,0,0)$ $\quad \underline{d(P_\pi,P)=1/3}$

$M=\vec{n}_\pi(A,B,C)$ $\quad \vec{u}_\pi(C,0,-A)$ $\quad \vec{v}_\pi(0,C,-B)$



Los puntos P_1 y P_2 están a 1/3 de π, luego lo que nos piden son las proyecciones de P_1 sobre y P_2 sobre π.

Para $P_1(0,0,0)$

· $r\perp\pi$ pasando por $P_1 \Rightarrow \begin{cases} x=2\lambda\\ y=-\lambda\\ z=2\lambda\end{cases}$

· $Q_1=r\cap\pi = 2\cdot 2\lambda+\lambda+2\cdot 2\lambda+1=0$

$9\lambda+1=0 \Rightarrow \boxed{\lambda=-1/9}$

$\Rightarrow Q_1(-2/9,\ 1/9,\ -2/9)$

Para $P_2(-2/5,2/5,-2/5)$

– $r\perp\pi$ pasando por $P_2 \Rightarrow \begin{cases} x=-2/5+2\lambda\\ y=2/5-\lambda\\ z=-2/5+2\lambda\end{cases}$

– $Q_2=r\cap\pi \Rightarrow -\frac{4}{5}+4\lambda-\frac{2}{5}+\lambda-\frac{4}{5}+4\lambda+1=0$

$9\lambda-1=0 \Rightarrow \lambda=1/9$

$Q_2(-8/45,\ 13/45,\ -8/45)$

44 A(−1, 3, −1), B(−3, 1, −7), C(0, 5, 1)
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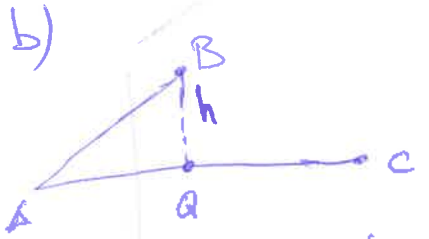
a) Son vértices de un triángulo
b) Hallar la long. del segmento que determina el pto B y su proyección sobre AC ("La long. de la altura")

a) $\vec{AB}\,(-2,-2,-6)$ $\quad \frac{-2}{1} \neq \frac{-2}{2} \neq \frac{-6}{2} \Rightarrow$ no paralelos $\Rightarrow$
$\vec{AC}\,(1,2,2)$ $\quad \Rightarrow$ no alineados $\Rightarrow$ ABC triángulo.

b) 

$Q = proy_{\vec{AC}} B$ y $h = d(B, Q)$

Construimos el plano $\pi \perp \vec{AC} \iff \vec{n}_\pi = \vec{AC} = (1,2,2)$
$\Rightarrow \pi \equiv x+2y+2z+D=0$ hacemos que pase por B
$-3+2-14+D=0 \Rightarrow D=15 \Rightarrow \pi \equiv x+2y+2z+15=0$

$Q = \pi \cap r_{AB}$ $\qquad r_{AB} \equiv \begin{cases} x = -1+\lambda \\ y = 3+2\lambda \\ z = -1+2\lambda \end{cases} \quad \lambda \in \mathbb{R}$

$Q \equiv -1+\lambda+2(3+2\lambda)+2(-1+2\lambda)+15=0 \Rightarrow \lambda=-2$
$\Rightarrow Q(-3,-1,-5) \Rightarrow h = |\vec{BQ}| = |(0,-2,2)| = \sqrt{8} = 2\sqrt{2}$ u

Hacemos el paralelogramo

La long. que nos piden es $h = \frac{\text{Área}}{\text{base}}$

$A = |\vec{AB} \wedge \vec{AC}| = \left| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -2 & -2 & -6 \\ 1 & 2 & 2 \end{vmatrix} \right| = |(8,2,2)| = \sqrt{72}$

$\text{base} = |\vec{AC}| = \sqrt{1+2^2+2^2} = 3$

$h = \frac{A}{\text{base}} = \frac{\sqrt{72}}{3} = \sqrt{8} = 2\sqrt{2}$ u

$\boxed{45}$ Esta en una hoja al principio

1] A(1,5,-2) B(4,0,1) C(-3,2,0)

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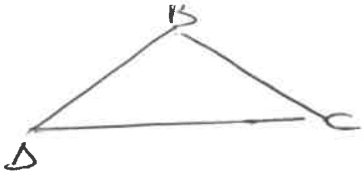


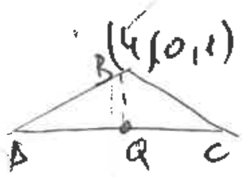
¿Son vértices de un triángulo?

b) Hallar long. del segmento que determina B y su proyección sobre AC. ("La long. de la altura")

a) $\vec{AB}(3,-5,3)$ $\qquad \frac{3}{-4} \neq \frac{-5}{-3} \neq \frac{3}{2} \Rightarrow$ no paralelos $\Rightarrow$ no alineados.

$\vec{AC}(-4,-3,2)$ $\qquad \Rightarrow$ ABC triángulo.

b)



$\vec{AC}(-4,-3,2)$

$r_{AB} \equiv \begin{cases} x = 1 - 4\lambda \\ y = 5 - 3\lambda \\ z = -2 + 2\lambda \end{cases}$

$\pi \perp r_{AB}$ pasando por B

16
-2

$-4(x-4) - 3y + 2(z-1) = 0$

$-4x - 3y + 2z + 14 = 0$

$\boxed{\pi \equiv 4x + 3y - 2z - 14 = 0}$

$Q = r_{AB} \cap \pi \Rightarrow$

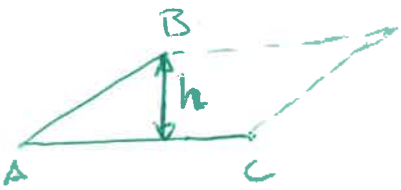
$\Rightarrow 4(1-4\lambda) + 3(5-3\lambda) - 2(-2+2\lambda) - 14 = 0$

$4 - 16\lambda + 15 - 9\lambda + 4 - 4\lambda - 14 =$

$= -29\lambda + 9 = 0 \Rightarrow \lambda = 9/29$

$Q\left(-7/29, \frac{118}{29}, \frac{-40}{29}\right)$

$d(B,Q) = \sqrt{\frac{1166}{29}}$



Hacemos el paralelogramo

La long. que nos piden es h

$A = |\vec{AB} \wedge \vec{AC}| = \left| \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 3 & -5 & 3 \\ -4 & -3 & 2 \end{vmatrix} \right| = |(-1,18,29)| = \sqrt{1166}$

base $= |\vec{AC}| = \sqrt{16+9+4} = \sqrt{29}$

$\boxed{h = \frac{A}{base} = \sqrt{\frac{1166}{29}}\ u}$

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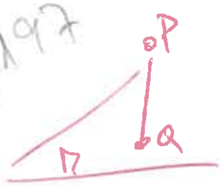

El pto más cercano es su proyección.

$r \perp \pi$ pasando por P.

$Q = r \cap \pi$

$Q(5,1,2)$

$d(P,\pi) = d(P,Q) = \sqrt{8}$

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$r \equiv \frac{x-1}{1} = \frac{y+1}{2} = \frac{z-2}{1}$ $\quad \vec{u}_r(1,2,1) \quad P_r(1,-1,2)$

$s \equiv \begin{cases} x - y + z = 2 \\ 3x - y - z = -4 \end{cases}$ $\quad \vec{u}_s = \left| \quad \right| =$

P.r → rectas paralelas

$\text{Área} = d(r,s)^2 = \frac{10}{3}$

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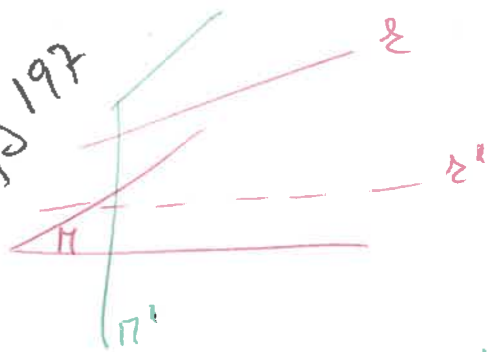


$\pi' \perp \pi \iff \vec{u}_{\pi'} = \vec{n}_{\pi}$

$\pi' \supset r \rightarrow \vec{v}_{\pi'} = \vec{u}_r \quad P_r \in \pi$

$$\pi' \equiv \begin{vmatrix} x-1 & y-1 & z-2 \\ 2 & 1 & 2 \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$\pi' \equiv 8x - 2y - 7z + 8 = 0$

$$r' = \begin{cases} 8x - 2y - 7z + 8 = 0 \\ x - 3y + 2z + 12 = 0 \end{cases}$$

47 Se consideran los ptos $P(2,1,-1)$, $Q(1,4,1)$ y $R(1,3,1)$

a) Comprueba que no están alineados y halla el área del triángulo que determinan

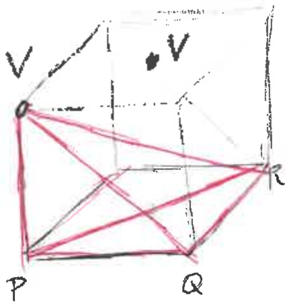
b) Si desde el pto $V(1,1,-1)$ rectas a cada uno de los ptos P, Q y R, se obtiene una pirámide. Halla la altura de dicha pirámide y su volumen.

a) $\vec{PQ}(-1,3,2)$ $\vec{PR}(-1,2,2) \rightarrow \frac{-1}{-1} \neq \frac{3}{2} \neq \frac{2}{2} \Rightarrow \vec{PQ} \nparallel \vec{PR} \Rightarrow$

$\Rightarrow P, Q, R$ no alineados.

$$A_{\widehat{PQR}} = \frac{1}{2}\left|\vec{PQ} \wedge \vec{PR}\right| = \frac{1}{2}\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 3 & 2 \\ -1 & 2 & 2 \end{vmatrix}\right| = \frac{1}{2}\left|(2,0,1)\right| = \boxed{\frac{\sqrt{5}}{2}\ u^2}$$

b)



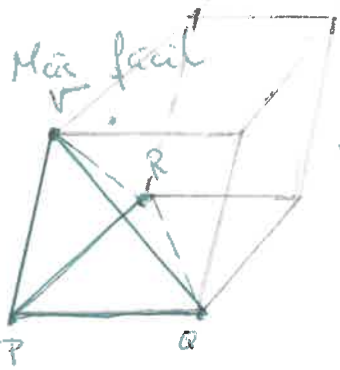
$h = d(V, \pi_{PQR})$

$$\pi \equiv \begin{vmatrix} x-2 & y-1 & z+1 \\ -1 & 3 & 2 \\ -1 & 2 & 2 \end{vmatrix} = 0 \Rightarrow \pi \equiv 2x - z - 3 = 0$$

$$\boxed{h = \frac{|2\cdot 1 - 1 - 3|}{\sqrt{4+1}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}\ u}$$

$$\boxed{V = \frac{1}{3} A_b \cdot h = \frac{1}{3} A_{\widehat{PQR}} \cdot h = \frac{1}{3} \cdot \frac{\sqrt{5}}{2} \cdot \frac{2\sqrt{5}}{5} = \frac{1}{3}\ u^3}$$

b) Más fácil



$$V_{pirámide} = \frac{1}{6} V_{paralelepípedo} = \frac{1}{6}\left|\vec{PQ} \cdot (\vec{PR} \wedge \vec{PV})\right| =$$

$$= \frac{1}{6}\left|\begin{vmatrix} -1 & 3 & 2 \\ -1 & 2 & 2 \\ -1 & 0 & 0 \end{vmatrix}\right| = \frac{1}{6} \cdot 2 = \frac{1}{3}\ u^3$$

$\vec{PV}(-1,0,0)$

$$V_{pirám} = A_b\, h \Rightarrow h = \frac{V_{pirám}}{A_b} = \frac{1/3}{\sqrt{5}/2} = \frac{2\sqrt{5}}{5}\ u$$

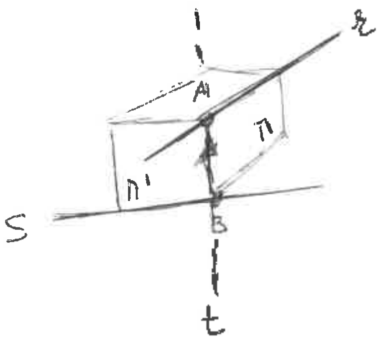
$\boxed{51}$ Consideramos las rectas $r \equiv \frac{x-3}{2} = y = \frac{z-1}{1}$; $s \equiv \begin{cases} y = -\mu \\ z = -\mu \end{cases}$ $\mu \in \mathbb{R}$

Hallar los pts que dan la mínima distancia y determina la ecuac. de la perpendicular común.

$\vec{u}_r\,(2,1,1)$ $\quad P_r\,(3,0,1)$ $\quad \vec{u}_s\,(1,-1,-1)$ $\quad P_s\,(0,0,0)$

$\vec{u}_r \not\parallel \vec{u}_s \Rightarrow$ Se cortan o se cruzan $\quad \overrightarrow{P_rP_s}\,(3,0,1) \rightarrow \begin{vmatrix} 2 & 1 & 1 \\ 1 & -1 & -1 \\ 3 & 0 & 1 \end{vmatrix} \neq 0$

$\Rightarrow$ Se cruzan



Perpendicular común: $t = \Pi \cap \Pi'$

$\Pi \equiv \begin{cases} r \subset \Pi \Rightarrow \vec{u}_r = \vec{u}_\Pi \quad P_r \in \Pi \\ t \subset \Pi \Rightarrow \vec{u}_t = \vec{v}_\Pi \end{cases} \Rightarrow \Pi \equiv \begin{vmatrix} x-3 & y & z-1 \\ 2 & 1 & 1 \\ 0 & 1 & -1 \end{vmatrix} = 0$

$\Pi' \equiv \begin{cases} s \subset \Pi' \\ t \subset \Pi' \end{cases} \Rightarrow \Pi' \equiv \begin{vmatrix} x & y & z \\ 1 & -1 & -1 \\ 0 & 1 & -1 \end{vmatrix} = 0$

$\Rightarrow \underline{\underline{t = \Pi \cap \Pi'}}$

$\left.\begin{array}{l} t \perp r \\ t \perp s \end{array}\right\} \Rightarrow \vec{u}_t = \vec{u}_r \wedge \vec{u}_s =$

$= \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 2 & 1 & 1 \\ 1 & -1 & -1 \end{vmatrix} = 3\vec{j} - 3\vec{k} \Rightarrow \vec{u}_t\,(0,1,-1)$

$A = r \cap \Pi'$

$B = s \cap \Pi$

Pts genéricos. $A \in r \Rightarrow A\,(3+2\lambda, \lambda, 1+\lambda)$ $\quad B \in s \Rightarrow B\,(\mu, -\mu, -\mu)$

$\overrightarrow{AB}\,(-3-2\lambda+\mu, -\lambda-\mu, -1-\lambda-\mu)$

$\overrightarrow{AB} \perp r \Rightarrow \overrightarrow{AB}\cdot\vec{u}_r = 0 \Rightarrow 6\lambda - 7 = 0 \Rightarrow \lambda = -7/6 \Rightarrow A\,\left(\frac{2}{3}, -\frac{7}{6}, -\frac{1}{6}\right)$

$\overrightarrow{AB} \perp s \Rightarrow \overrightarrow{AB}\cdot\vec{u}_s = 0 \Rightarrow 2 + 3\mu = 0 \Rightarrow \mu = 2/3 \Rightarrow B\,\left(2/3, -2/3, -2/3\right)$

t recta que pasa por A y $B \Rightarrow \overrightarrow{AB}\,\left(0, 1/2, -1/2\right) \approx (0,1,-1)$

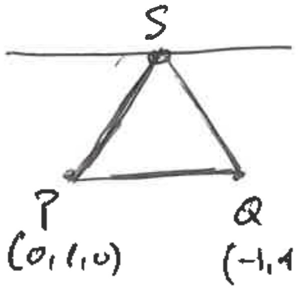
$$t \equiv \begin{cases} x = -2/3 \\ y = -7/6 + \lambda \\ z = -1/6 - \lambda \end{cases} \quad \lambda \in \mathbb{R}$$

52 $P(0,1,0)$ y $Q(-1,1,1)$ son dos vértices de un triángulo y el tercero, S, pertenece a la recta $r \equiv \begin{cases} x=4 \\ z=1 \end{cases}$.
La recta que contiene a P y S es $\perp$ a la recta r.

a) Determinar S

b) Calcular el área del triángulo $\widehat{PQS}$.

a)



$S(4, \lambda, 1)$ pto genérico de r

$r_{\vec{PS}} \perp r \iff \vec{PS} \perp \vec{u}_r \iff \vec{PS}\cdot\vec{u}_r = 0$

$\left.\begin{array}{l} \vec{PS}(-4, 1-\lambda, -1) \\ \vec{u}_r(0,1,0) \end{array}\right\} (-4, 1-\lambda, -1)(0,1,0) = 0 \Rightarrow \underline{\underline{\lambda = 1}}$

$\Rightarrow \underline{S(4,1,1)}$

b) $A_{\widehat{PQS}} = \dfrac{1}{2}\,|\vec{PQ}\wedge\vec{PS}| = \dfrac{1}{2}\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ -1 & 0 & 1 \\ -4 & 0 & -1 \end{vmatrix}\right| = \dfrac{1}{2}\cdot 5 = \boxed{\dfrac{5}{2}\,u^2}$

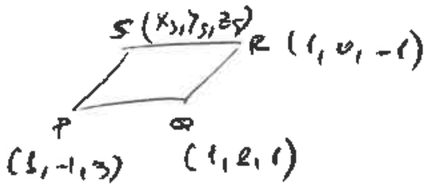
56 a) Hallar la distancia del pto $P(1,-1,3)$ a la recta $r_{\vec{QR}}$ con $Q(1,2,1)$ y $R(1,0,-1)$

b) Hallar todos los ptos S del plano determinado por P, Q, R, tal que P, Q, R, S determinen un paralelog.

a) $d(P, r_{\vec{QR}}) = \dfrac{|\vec{PR}\wedge\vec{QR}|}{|\vec{QR}|} = \dfrac{\left|\begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ 0 & 1 & -4 \\ 0 & -2 & -2 \end{vmatrix}\right|}{|(0,-2,-2)|} = \dfrac{10}{\sqrt{8}} = \dfrac{5}{\sqrt{2}} = \underline{\underline{\dfrac{5\sqrt{2}}{2}\,u}}$

b) I) Si consideramos que los vértices del paralelogramo tienen que seguir el orden PQRS

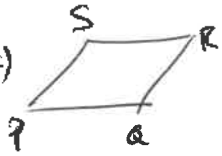
$\vec{PQ} \approx \vec{SR}$



$(0,3,-2) = (1-x_s, -y_s, -1-z_s) \Rightarrow \begin{array}{l} x_s = 1 \\ y_s = -3 \\ z_s = 1 \end{array} \quad \boxed{S(1,-3,1)}$

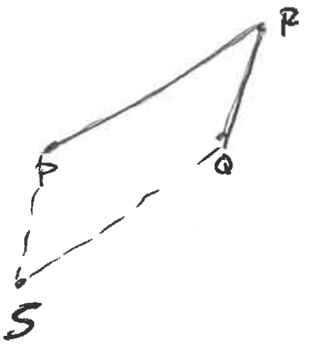
II) Si consideramos que pueden estar en otro orden hay 3 posibilidades.

a)



b)

c)



a) Ya está resuelto

b) $\vec{PQ} = \vec{RS}$

$$(0,3,-2) = (x_s-1,\ y_s,\ z_s+1) \Rightarrow \begin{cases} x_s = 1 \\ y_s = 3 \\ z_s = -3 \end{cases} \qquad \boxed{S(1,3,-3)}$$

c) $\vec{PR} = \vec{SQ}$

$$(0,1,-4) = (1-x_s,\ 2-y_s,\ 1-z_s) \Rightarrow \begin{cases} x_s = 1 \\ y_s = 1 \\ z_s = 5 \end{cases} \qquad \boxed{S'(1,1,5)}$$

57 Hallar el plano de la familia $mx+y+z-(m+1)=0$ que está situado a 1 u del origen de coordenadas.

$$d(O,\pi)=\frac{|m+1|}{\sqrt{m^2+1+1}}=\frac{|m+1|}{\sqrt{m^2+2}}=1 \Rightarrow |m+1|=\sqrt{m^2+2}$$

$$\Rightarrow m^2+2m+1=m^2+2 \Rightarrow 2m=1 \Rightarrow \boxed{m=1/2} \Rightarrow \pi\equiv \frac{1}{2}x+y+z-\frac{3}{2}=0$$

$$\Rightarrow \pi\equiv x+2y+2z-3=0$$

59 a) a? b? para que $\begin{cases} x+ay+z=0 \\ x+y-2z=b \\ -2x+y+z=-2 \end{cases}$ se corten en una recta.

b) Hallar el sim. de $O(0,0,0)$ respecto de esa recta.

a) para que se corten en una recta $\Rightarrow$ S.C.I (dep. de 1 param)

($\Rightarrow$ ran A = ran A^* $=2<n=3$)

$$A^*=\left(\begin{array}{ccc|c} 1 & a & 1 & 0 \\ 1 & 1 & -2 & b \\ -2 & 1 & 1 & -2 \end{array}\right) \rightarrow |A|=\begin{vmatrix} 1 & a & 1 \\ 1 & 1 & -2 \\ -2 & 1 & 1 \end{vmatrix}=0 \Rightarrow 3a+6=0 \Rightarrow a=-2$$

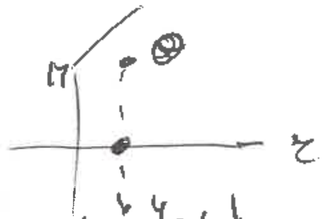
$$\Rightarrow \text{rang}(A)\neq 3 \quad \text{como} \quad \begin{vmatrix} 1 & -2 \\ -2 & 1 \end{vmatrix}\neq 0 \Rightarrow \text{rang}(A)=2$$

Vamos ran A^* $\rightarrow$ orlando $\rightarrow$ $|A|$

$$\begin{vmatrix} 1 & 1 & 0 \\ 1 & -2 & b \\ -2 & 1 & -2 \end{vmatrix}=0 \Rightarrow b=2$$

Luego si $a=-2$ y $b=2$ $\Rightarrow$ ran A = ran A^* $=2<n=3$ $\Rightarrow$ se cortan en una recta $\Rightarrow$ determinado por $r\equiv\begin{cases} x-2y+z=0 \\ x+y-2z=2 \end{cases}$

b)



i) $\pi\perp r$, $O\in\pi$ $\Rightarrow \pi\equiv x+y+z=0$

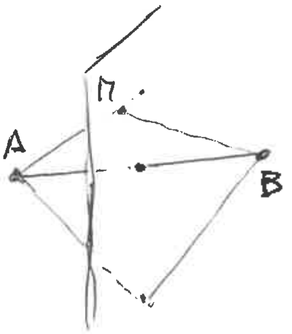
$$r\equiv\begin{cases} x=\frac{4}{3}+\lambda \\ y=\frac{2}{3}+\lambda \quad \lambda\in\mathbb{R} \\ z=\lambda \end{cases}$$

ii) $Q=r\cap\pi \Rightarrow \frac{4}{3}+\lambda+\frac{2}{3}+\lambda+\lambda=0 \Rightarrow \lambda=-2/3$

$\Rightarrow Q(2/3, 0, -2/3)$

iii) Q es el pto medio de $\overrightarrow{OO'}$ $\Rightarrow O'\left(\frac{4}{3}, 0, -\frac{4}{3}\right)$

61 "Plano mediador". Hallar el l. g. de los pts $M(x,y,z)$ que equidistan de los pts $A(1,-1,0)$ y $B(2,3,-4)$.



$d(P,A) = d(P,B) \Rightarrow$

$\Rightarrow \sqrt{(x-1)^2+(y+1)^2+z^2} = \sqrt{(x-2)^2+(y-3)^2+(z+4)^2} \Rightarrow$

$(x-1)^2+(y+1)^2+z^2 = (x-2)^2+(y-3)^2+(z+4)^2 \Rightarrow$

$\Rightarrow 2x+8y-8z-27=0 \rightarrow$ PLANO

Este plano recibe el nombre de plano mediador, y es un plano $\perp \vec{AB}$, que pasa por su pto medio.

$P_{me}(\vec{AB}) = \left(\frac{1+2}{2}, \frac{-1+3}{2}, \frac{0-4}{2}\right) = \left(\frac{3}{2}, 1, -2\right)$

$\Pi \perp \vec{AB} \Leftrightarrow \vec{n}_\Pi = \vec{AB} = (1,4,-4) \Rightarrow \Pi \equiv x+4y-4z+D=0$

pasa por $P_{me}(\vec{AB}) \Rightarrow \frac{3}{2}+4-4(-2)+D=0 \Rightarrow D=-27/2$

$\Rightarrow \Pi \equiv x+4y-4z-\frac{27}{2}=0 \Leftrightarrow \Pi \equiv 2x+8y-8z-27=0$